

What is $(999 - 99 + 9) \div 9$?

We have $(999 - 99 + 9) \div 9 = 909 \div 9 = 101$

How many minutes are there in $\frac{1}{12}$ of a day?

1 day = 24 hours

$$\Rightarrow \frac{1}{12} \text{ day} = 2 \text{ hours}$$

= 120 min

In my row in the theatre the seats are numbered consecutively From T1 to T50. I am sitting in T17 and you are sitting in T39. How many seats are there between us?

There are $(39 - 17) - 1 = 21$ seats between us

The number 987654321 is multiplied by 9. How many times does the digit 8 occur in the result? (No calc)

For any number n we have $9n = 10n - n$ so to

Find 987654321×9 we calculate

$$\begin{array}{cccccccccc}
 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
 9 & 17 & 16 & 15 & 14 & 13 & 12 & 11 & 10 & 9 \\
 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & +1 & 0 \\
 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
 \hline
 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 8 & 9
 \end{array}$$

So the answer is 9

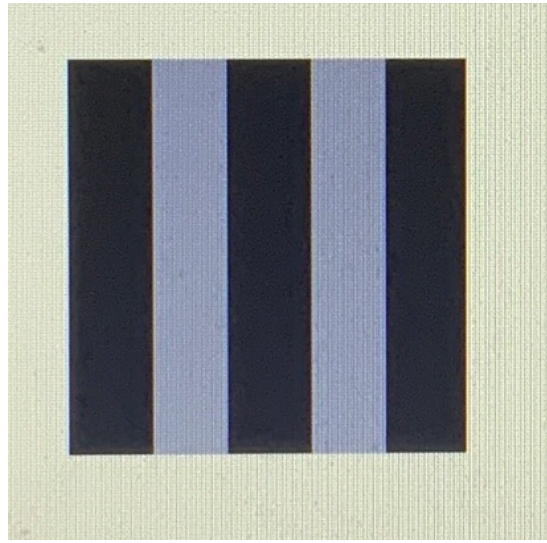
What is the difference between the smallest 4-digit number and the largest 3-digit number?

The smallest 4-digit number is 1000.

The largest 3-digit number is 999.

The difference is 1

The diagram shows a square divided into strips of equal width. Three strips are black and two are grey. What Fraction of the perimeter of the square is grey?



Suppose the side length of the square is $5x$.

The length of perimeter which is grey is $4x$.

The total perimeter is $4 \cdot 5x = 20x$.

So the Fraction which is grey is $\frac{4x}{20x} = \frac{1}{5}$

What is $2014 - 4102$?

We have $2014 - 4102 = 2002 - 4102 + 12$

$$= -2100 + 12$$

$$= -2088$$

How many prime numbers are there in the list?

1, 12, 123, 1234, 12345, 123456

1 is neither prime nor composite

12 is composite

123 has digit sum 6 so is composite (multiple of 3)

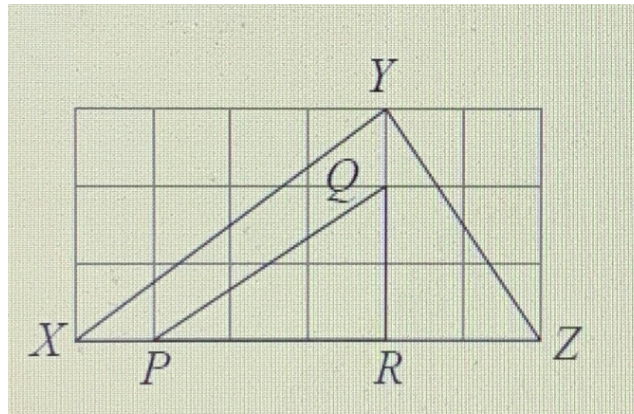
1234 is even and hence composite

12345 is a multiple of 5

123456 is even and hence composite

So the answer is 0

Triangles XYZ and PQR are drawn on a square grid.
What Fraction of the area of triangle XYZ is the
area of triangle PQR?



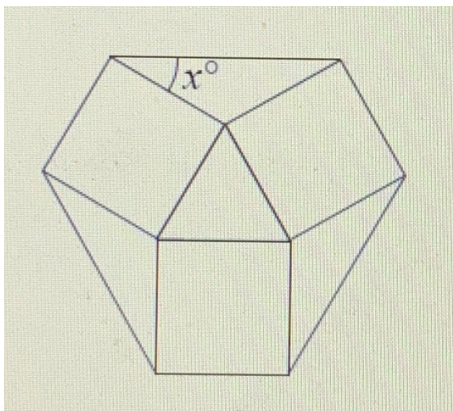
Suppose each box in the grid has side length x
Then

$$[PQR] = \frac{1}{2} \cdot 2x \cdot 3x = 3x^2$$

$$[XYZ] = \frac{1}{2} \cdot 3x \cdot 6x = 9x^2$$

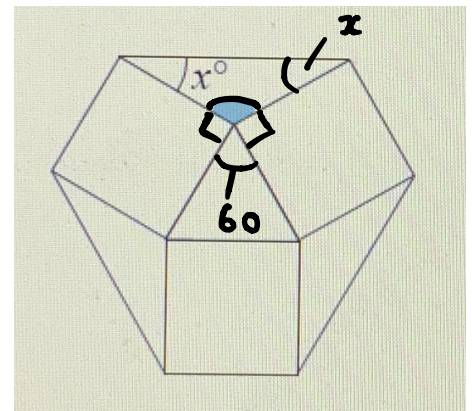
$$\text{So } \frac{[PQR]}{[XYZ]} = \frac{1}{3}$$

An equilateral triangle is surrounded by three squares
as shown. Find x



By angle sum at a
point, the blue angle is
 $360 - 60 - 2 \times 90$
 $= 120^\circ$

By angle sum of a
triangle:



$$2x + 120 = 180$$

$$\Rightarrow x = 30^\circ$$

The First two terms of a sequence are 1 and 2. Each of the Following terms in the sequence is the sum of all the terms which come before it in the sequence. Which of these is not a term in the sequence?

A) 6 B) 24 C) 48 D) 72 E) 96

Start calculating terms:

1, 2, 3, 6, 12, 24, 48, 96, ...

Each term after the third is double the previous.

The answer is therefore **72**

In this subtraction, P, Q, R, S, T represent single digits. What is the value of $P+Q+R+S+T$?

$$\begin{array}{r} 7 \ Q \ 2 \ 5 \ T \\ P \ 3 \ R \ 9 \ 6 \ - \\ \hline 2 \ 2 \ 2 \ 2 \ 2 \end{array}$$

Rewrite as addition:

$$\begin{array}{r} 1 1 \\ P \ 3 \ R \ 9 \ 6 \\ 2 \ 2 \ 2 \ 2 \ 2 \ + \\ \hline 7 \ Q \ 2 \ 5 \ T \end{array}$$

From the units column: $T = 6 + 2 = 8$

From the tens column: $9 + 2 = 11$, so $S = 1$ and there is a carry into the hundreds column

From the hundreds column: $1 + R + 2 \equiv 2 \pmod{10}$
 $\Rightarrow R = 9$

and there is a carry into the thousands column

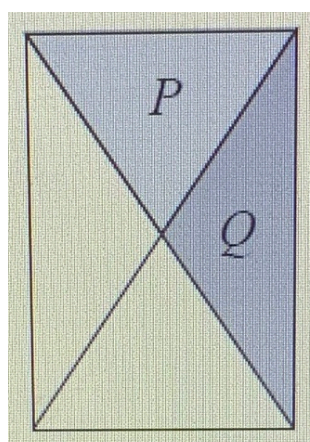
From the thousands column: $1+3+2 = Q \Rightarrow Q = 6$

and there is no carry

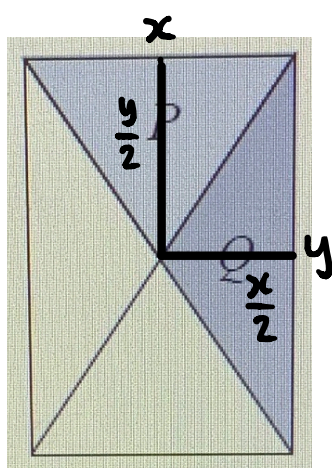
From the 10 000 column: $P+2 = 7 \Rightarrow P = 5$

So $P+Q+R+S+T = 5+6+9+1+8 = 29$

A rectangle is split into triangles by drawing in its diagonals. What is the ratio of the area of triangle P to the area of triangle Q?



Suppose the shorter side has length x and the longer side has length y . Then by symmetry, the height of triangle P is $y/2$ and the height of triangle Q is $x/2$.



So

$$\text{Area } P = \frac{1}{2} \cdot x \cdot \frac{y}{2} = \frac{xy}{4}$$

$$\text{Area } Q = \frac{1}{2} \cdot \frac{x}{2} \cdot y = \frac{xy}{4}$$

Hence the ratio is 1:1

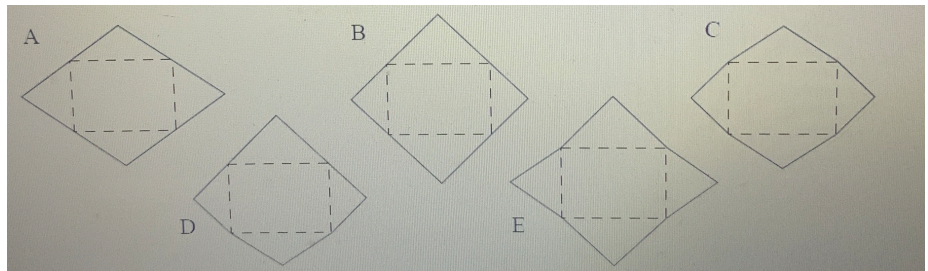
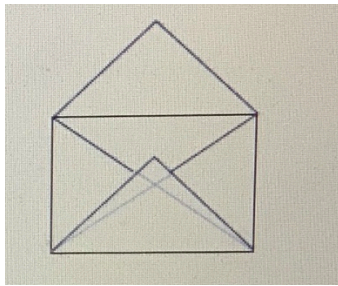
Which of these is equal to One million millimetres?

A) 1 metre B) 10 metres C) 100 metres

D) 1 kilometre E) 10 kilometres

$$\begin{aligned} 10^6 \text{ millimetres} &= 10^5 \text{ centimetres} \\ &= 10^3 \text{ metres} \\ &= 1 \text{ kilometre} \end{aligned}$$

The diagram shows a rectangular envelope made by folding a single piece of paper. What could the original unfolded piece of paper look like?



The top and bottom triangles must be congruent.

There should be an angle less than 180° not covered by paper at the corners where the triangles meet.

So the answer is **E**

Only one of the following statements is true.
Which one?

- A) B is true B) E is False C) all are true
D) all are false E) A is False

Since C and D are mutually exclusive, neither can be true.

If A is true: then B is true, but we can't have

two true statements.

If B is true: then E is False, so A is true.

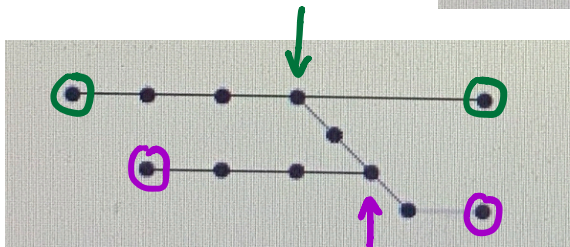
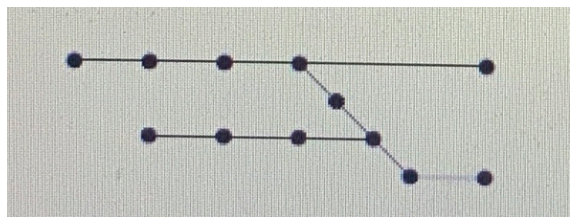
But we can't have two true statements

If E is true: then A is False, and B is False, and so E is true. This works.

So the answer is **E**

The diagram is a map of Jo's local rail network.

Dots represent stations and lines are routes. Jo wants to visit all the stations. What is the smallest number of stations she must go to more than once?



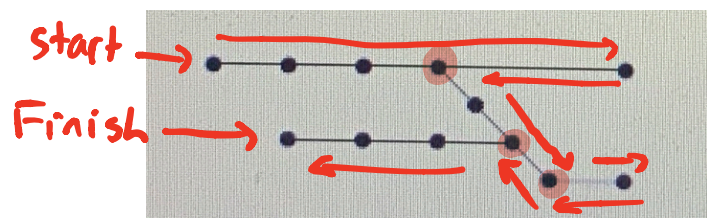
It is impossible to visit the two stations circled in green without passing through the indicated (arrow)

dot more than once. Even if you start or Finish there, you have to go through that station to access the rest of the rail line. A similar argument applies to the stations indicated in purple.

So she must pass through at least 2 stations twice.

After visiting a circled station of one colour, she must retrace her steps to visit the other circled station of that colour. To minimise the number of stations visited, she should only double back after visiting the rightmost stations since the path from the arrowed stations to them are shorter.

So the best path is as follows:



The number of stations she visits more than once is **3**

Which of these statements is true?

A) $15614 = 1 + 5^6 - 1 \times 4$ B) $15615 = 1 + 5^6 - 1 \times 5$

C) $15616 = 1 + 5^6 - 1 \times 6$ D) $15617 = 1 + 5^6 - 1 \times 7$

E) $15618 = 1 + 5^6 - 1 \times 8$

5^6 has units digit 5 so $1 + 5^6$ has units digit 6.
Find the units digit of all of the options:

$1 + 5^6 - 4$ ends in 2

$1 + 5^6 - 5$ ends in 1

$1 + 5^6 - 6$ ends in 0

$1 + 5^6 - 7$ ends in 9

$1 + 5^6 - 8$ ends in 8

So the answer must be **E**

Jack and Jill played a game for 2 people. In each game, the winner was awarded 2 points and the loser 1 point. No games were drawn. Jack won exactly 4 games and Jill had a Final score of 10 points. How many games did they play?

Since Jack won 4 games exactly, Jill scored 4 points from losing those games. So she scored 6 points from winning games and therefore won 3. Hence the total number of games they played was $4 + 3 = \mathbf{7}$

Box P has p chocolates and box Q has q chocolates. p and q are both odd and $p > q$. What is the smallest number of chocolates that would have to be moved from box P to box Q so that box Q has more chocolates than box P?

Since $p+q$, the total number of chocolates, is even, it must be that after transferring any number of chocolates between the boxes, either both have an odd number or both have an even number.

So if Q has more chocolates than P after the transfer, the minimum difference between them is 2.

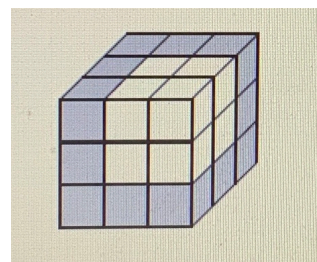
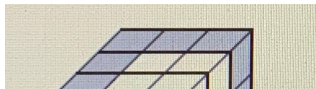
Let k be the number of chocolates transferred.
We have

$$p - k + 2 = q + k$$

$$\Rightarrow 2k = p - q + 2$$

$$\Rightarrow k = \frac{p - q + 2}{2}$$

Pablo's teacher has given him 27 identical white cubes. She asks him to paint some of the Faces of these cubes grey and then stack the cubes so that they appear as shown. What is the largest possible number of cubes which Pablo can leave with no Faces painted grey?



On the bottom layer, there must be $S+1$ cubes with at least one

grey Face.

On the middle layer, there must be 2+ cubes with at least one grey Face.

On the top layer, there must be 5+ cubes with at least one grey Face.

So there must be at least $5+2+5 = 12$ cubes which are painted, and therefore the answer is

$$27 - 12 = 15$$

In the division calculation $952473 \div 18$, which two adjacent digits should be swapped in order to increase the result by 100? (NO CALC)

$$\begin{array}{cccccc} 9 & 5 & 2 & 4 & 7 & 3 \\ \hline & \square & \square & \square & \square & \square \\ & A & B & C & D & E \end{array}$$

Swaps A, B and E decrease the number, so the result of the division will also decrease.

Swap D will increase the number by $743 - 473 = 270$ so the result of the division will increase by $270 \div 18$, which is too small.

Swap C will increase the number by

$$4273 - 2473 = 1800$$

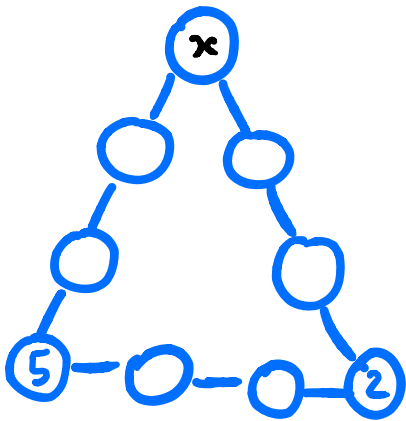
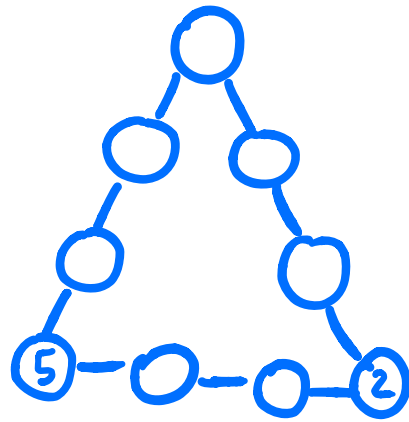
So the result of the division will increase by

$$1800 \div 18 = 100$$

Hence we should swap the 2 and 4

Sam wants to complete the diagram so that each

of the nine circles contains one of the digits from 1 to 9 inclusive and each contains a different digit. Also, the digits in each of the three lines of four circles must have the same total. What is this total?



Suppose the number at the top of the pyramid is x , and the total of each line is L .

Adding up all three lines, we obtain

$$(1 + \dots + 9) + 5 + 2 + x = 3L$$

$$\Rightarrow 45 + 7 + x = 3L$$

$$\Rightarrow 52 + x = 3L$$

We know that $x \leq 9$ and $x \neq 2, 5$.

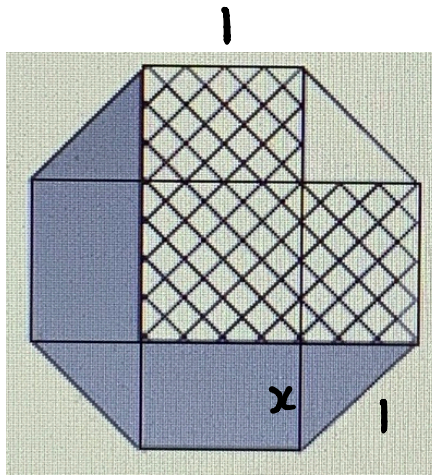
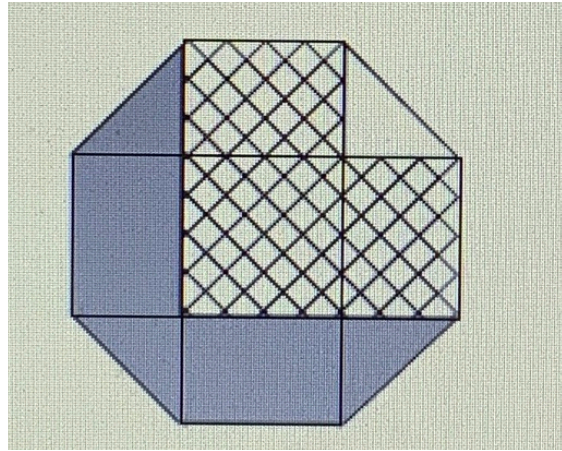
The LHS must be a multiple of 3, so the only options are

$$\begin{array}{lll} 52 + x = 54 & 52 + x = 57 & 52 + x = 60 \\ \Rightarrow x = 2 & \Rightarrow x = 5 & \Rightarrow x = 8 \quad \checkmark \end{array}$$

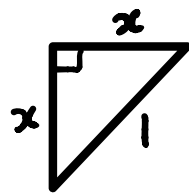
So $x = 8$ and hence

$$3L = 60 \Rightarrow L = 20$$

The diagram shows a regular octagon with sides of length 1. The octagon is divided into regions by 4 diagonals. What is the difference between the area of the hatched region and the area of the region shaded grey?



Let x be the length indicated in the diagram.



We have $2x^2 = 1$
 $\Rightarrow x^2 = \frac{1}{2}$
 $\Rightarrow x = \frac{1}{\sqrt{2}}$

The shaded region is comprised of 3 congruent triangles and two rectangles.

Each triangle has area $\frac{1}{2} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} = \frac{1}{4}$

Each rectangle has area $1 \cdot \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

So the total area is $3 \cdot \frac{1}{4} + 2 \cdot \frac{1}{\sqrt{2}} = \frac{3}{4} + \sqrt{2}$

The hatched region is comprised of 2 rectangles and one square.

As above, each rectangle has area $\frac{1}{\sqrt{2}}$

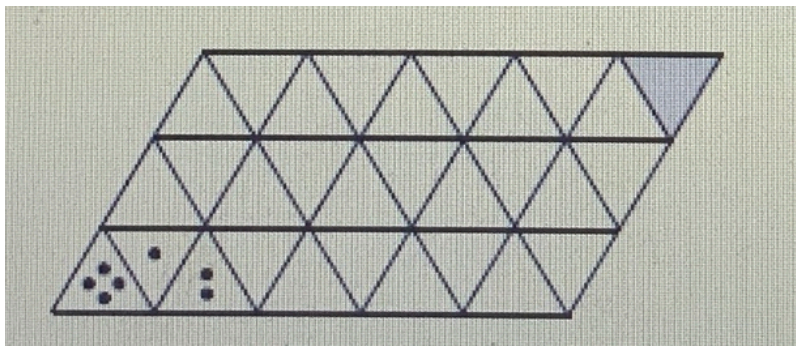
The Square has area $1 \times 1 = 1$.

So the total area is $1 + 2 \cdot \frac{1}{\sqrt{2}} = 1 + \sqrt{2}$

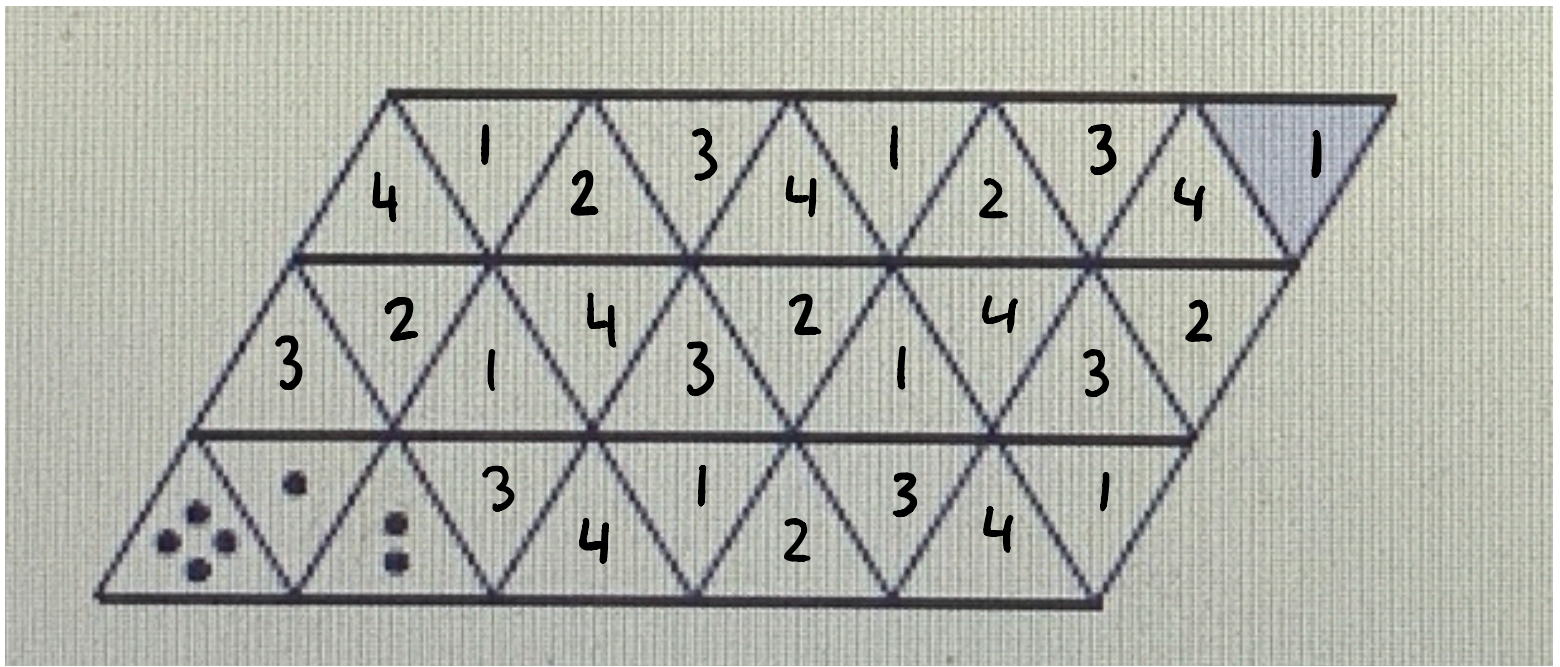
Hence the difference between the areas is

$$(1 + \sqrt{2}) - \left(\frac{3}{4} + \sqrt{2}\right) = \frac{1}{4}$$

A die has the shape of a regular tetrahedron, with the Four Faces having 1, 2, 3 and 4 pips. The die is placed with 4 pips 'Face down' in one corner of the triangular grid shown, so that the Face with 4 pips precisely covers the triangle marked with 4 pips. The die is now moved by rotating about an edge without slipping so that 1 pip is Face down. It is moved again so that 2 pips are face down. The rolling continues until the die rests on the shaded triangle in the opposite corner of the grid. How many pips are now Face down?



If the die is rolled around a single vertex, it covers 6 small triangles, making a hexagon shape. It uses 3 of the 4 Faces, repeated twice (it never lands on the face opposite the vertex). If we roll it away from the hexagon in any direction, it will land on the 4th Face. In this method, we can fill in all the triangles in the grid.



The three given tiles would be part of a hexagon using the 4, 1, 2 sides so when we roll one space to the right we land on the 3.

Now using the next three tiles, we have a hexagon using 1, 2, 3, etc.

So no matter which path we take, we will always land on a 1